Uniformly accelerated oscillator in 2+1 dimensions: temperature-dependent frequency shift

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1. The Unruh effect

Observers moving with constant acceleration of magnitude α in Minkowski spacetime perceive the Minkowski vacuum state as a thermal state at a temperature proportional to their acceleration, known as the Unruh temperature

$$T_U = \frac{\alpha}{2\pi}.$$

Accelerated observers through the Minkowski vacuum respond in exactly the same way as inertial observers immersed in a thermal reservoir at the Unruh temperature. This equivalence ceases to hold when the dimensions of the background spacetime are odd [1]. We ask:

What this non-equivalence in odd spacetime dimensions implies for the dynamics of an accelerated Brownian particle?

2. Setup

We consider a massless scalar quantum field on a background (n+1)-dimensional Minkowski spacetime

$$\hat{\Phi}(t, \mathbf{x}) = \int \frac{d^n \boldsymbol{k}}{\sqrt{2(2\pi)^n |\boldsymbol{k}|}} \left(\hat{a}_{\boldsymbol{k}} e^{-i(|\boldsymbol{k}|t - \boldsymbol{k}\cdot\mathbf{x})} + \text{H.c.} \right),$$

We consider an oscillator detector of unit mass and bare frequency Ω, whose position operator $\hat{x}$ is linearly coupled to the field through the interaction Hamiltonian [2]

$$H_{\text{int}} = \lambda \hat{x} \otimes \hat{\Phi}(\mathsf{x}(\tau)),$$

where λ is the coupling constant and $\hat{\Phi}(\mathsf{x}(\tau))$ is the pullback of the field to the detector's worldline $\mathsf{x}(\tau) = (t(\tau), \mathbf{x}(\tau))$ parametrized by its proper time τ.

A uniformly accelerated detector follows the hyperbolic trajectory

$$\mathsf{x}(\tau) = (\alpha^{-1}\sinh(\alpha\tau), \alpha^{-1}\cosh(\alpha\tau), x_\perp),$$

where $x_\perp$ denotes the spatial coordinate transverse to the direction of the acceleration.

3. Noise and dissipation kernels

The Wightman two-point correlation function of the field evaluated along a detector's trajectory can be written as

$$\mathcal{W}(\tau, \tau') = \langle \hat{\varphi}(\tau)\hat{\varphi}(\tau') \rangle \equiv \nu(\tau, \tau') - i\eta(\tau, \tau'),$$

where $\hat{\varphi}(\tau) := \lambda\hat{\Phi}(\mathsf{x}(\tau))$, and we have introduced the so-called *noise kernel*

$$\nu(\tau, \tau') := \frac{1}{2}\langle \{\hat{\varphi}(\tau), \hat{\varphi}(\tau')\} \rangle$$

and *dissipation kernel*

$$\eta(\tau, \tau') := \frac{i}{2}\left\langle [\hat{\varphi}(\tau), \hat{\varphi}(\tau')] \right\rangle.$$

4. The master equation

The reduced dynamics of the oscillator detector is descibed by the master equation

$$\frac{d}{d\tau}\hat{\rho}(\tau) = -i\left[\frac{\hat{p}^2}{2} + \frac{1}{2}(\Omega^2 + \delta\Omega)\hat{x}^2, \hat{\rho}(\tau)\right] - i\gamma[\hat{x}, \{\hat{p}, \hat{\rho}(\tau)\}] - D[\hat{x}, [\hat{x}, \hat{\rho}(\tau)]] - f[\hat{x}, [\hat{p}, \hat{\rho}(\tau)]],$$

where the coefficient

$$\gamma \equiv \frac{1}{\Omega}\int_0^\infty d\tau\, \eta(\tau)\sin(\Omega\tau)$$

is the dissipation rate and

$$\delta\Omega \equiv -2\int_0^\infty d\tau\, \eta(\tau)\cos(\Omega\tau)$$

is a shift to oscillator's frequency.

5. Two-point correlation functions

Spacetime dimensions	Oscillator detector	Wightman function
2+1	inertial in thermal field bath	$\mathcal{W}_{\text{th}}(t) = \frac{\lambda^2}{4\pi}\int_0^\infty d\omega \left(\coth\left(\frac{\omega}{2T}\right)\cos(\omega t) - i\sin(\omega t) \right)$
	accelerated in vacuum	$\mathcal{W}_{\text{acc}}(t) = \frac{\lambda^2}{4\pi}\int_0^\infty d\omega \left(\cos(\omega t) - i\tanh\left(\frac{\omega}{2T_U}\right)\sin(\omega t) \right)$
3+1	inertial in thermal field bath	$\mathcal{W}_{\text{th}}(t) = \frac{\lambda^2}{4\pi^2}\int_0^\infty d\omega\, \omega \left(\coth\left(\frac{\omega}{2T}\right)\cos(\omega t) - i\sin(\omega t) \right)$
	accelerated in vacuum	$\mathcal{W}_{\text{acc}}(t) = \frac{\lambda^2}{4\pi^2}\int_0^\infty d\omega\, \omega \left(\coth\left(\frac{\omega}{2T_U}\right)\cos(\omega t) - i\sin(\omega t) \right)$

6. Dissipation and frequency shift

We find that both the accelerated detector's dissipation rate

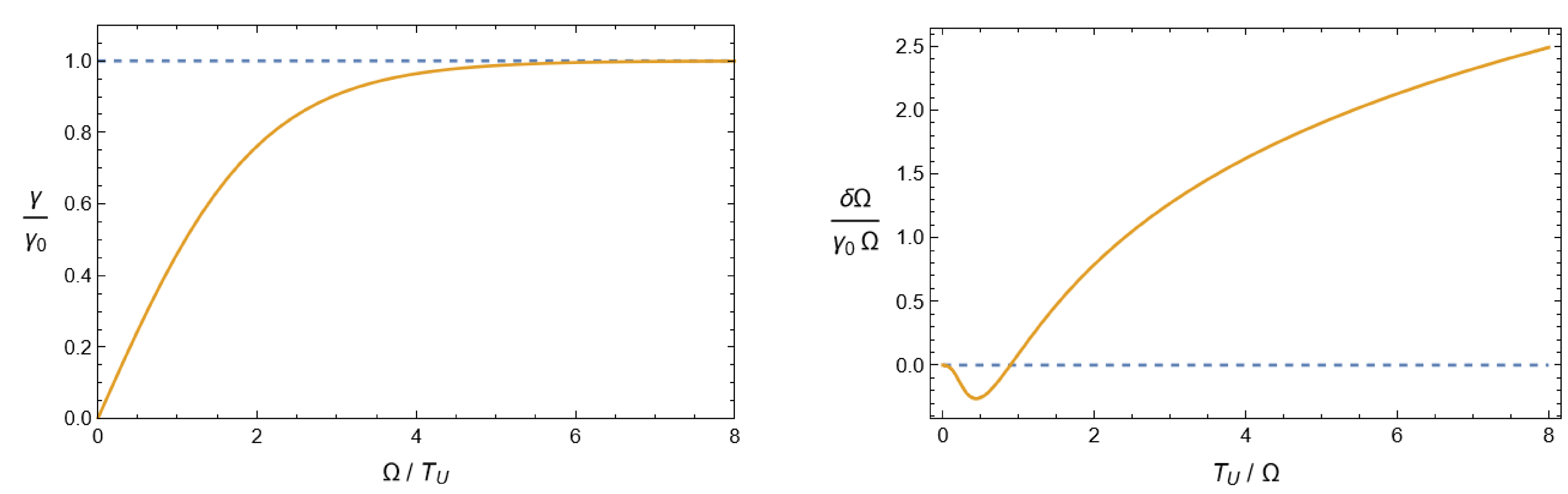
$$\gamma = \gamma_0 \tanh\left(\frac{\Omega}{2T_U}\right),$$

where $\gamma_0 = \frac{\lambda^2}{8\Omega}$ denotes the damping constant obtained in the case of inertial motion in field bath at a thermal state, and the shift of its frequency caused by the coupling to the field

$$\delta\Omega = \frac{4\gamma_0\Omega}{\pi}\left[\ln\left(\frac{2\pi T_U}{\Omega}\right) + \text{Re}\psi\left(\frac{i\Omega}{2\pi T_U} + \frac{1}{2}\right)\right]$$

depend on the acceleration temperature ; $\psi(z)$ is the digamma function.

This is in contrast to the (3+1)-dimensional case, where dissipation and frequency shift do not exhibit temperature dependencies.



(a) Dissipation rate (b) Frequency shift

Figure 1: Dissipation rate and frequency shift of a uniformly accelerated oscillator detector (solid line) in the (2+1)-dimensional space compared to the ones found in the case of an inertial detector immersed in a heat bath at the Unruh temperature T_U (dashed line).

7. Question

Can we explore the emergence of the unique characteristics of the Unruh (or any Unruh-like) effect in the (2+1) dimensional spacetime geometry in analogue experiments?

References

[1] S. Takagi, "Vacuum Noise and Stress Induced by Uniform Acceleration", Progress of Theoretical Physics Supplement 88, 1 (1986).

[2] B. L. Hu, S.-Y. Lin, and J. Louko, "Relativistic quantum information in detectors-field interactions", Class. Quantum Grav. 29, 224005 (2012).

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