Relational Dynamics and Time Travel: A Road Map

Sebastian Schuster¶, **Ana Alonso Serrano**†, **Jessica Santiago**§, **Matt Visser**‡

UNIVERZITA KARLOVA
Matematicko-fyzikální fakulta

¶: Ústav teoretické fyziky, Matematicko-fyzikální fakulta, Univerzita Karlova, Prague, †: Max-Planck-Institut für Gravitationsphysik, Potsdam, §: Section of Astrophysics, Astronomy and Mechanics, Aristotle University of Thessaloniki, ‡: School of Mathematics and Statistics, Victoria University Wellington

¶: sebastian.schuster@utf.mff.cuni.cz

Looking for me?

Do you want to ask questions or discuss, but I am not at my poster?
Please look for the person in the picture! ☺
(From the back, look for a long, low ponytail!)

Introduction and Methods

Canonical Quantum Gravity

Context:

- **Wheeler–DeWitt equation** of canonical quantum gravity from quantizing the Hamiltonian constraint of $3+1$-dimensional general relativity:
$$\hat{H}\,|\Psi\rangle = 0$$
- Usually, configuration space is the set of three-geometries over a fixed 3-manifold M_3.
- Issue: **Problem of time**, as equation is of the form of a time-independent Schrödinger equation for energy 0
- **No parameter time**, but (sub)systems can have **relational/emergent time**

Use **Page–Wootters (PW) formalism:** [PW83]:

- Essentially:
  1. Separate full Hilbert space: $\mathcal{H} = \mathcal{H}_{\mathrm{C}} \otimes \mathcal{H}_{\mathrm{R}}$
  2. Introduce a clock Hamiltonian $\hat{H}_{\mathrm{C}}$ of a subsystem
  3. Fix the clock state ψ_C at some chosen, initial time
  4. Define time through evolution of this state with $\hat{H}_{\mathrm{C}}$
  5. Measure time evolution of an operator $\hat{A}$, stationary w.r.t. $\hat{H}_{\mathrm{C}}$, as
  $$E(A|\tau) = \mathrm{tr}\left(\hat{A}\hat{P}_\tau\hat{\rho}\right) / \,\mathrm{tr}\left(\hat{P}_\tau\hat{\rho}\right),$$
  where
  $$\hat{P}_\tau = |\psi_C(\tau)\rangle\,\langle\psi_C(\tau)| \otimes \mathbb{1}_{\mathrm{R}}, \qquad \text{and} \qquad \hat{\rho} \in \mathcal{L}(\mathcal{H})$$
- Early counterarguments have recently been tackled, providing a unified picture, with clocks as a gauge to be chosen.[HSL21; GLM15; MV17]

Time Operators & POVMs

- **No self-adjoint (time) operators**, canonically conjugate to semi-bounded Hamiltonian generating time-shift group
- $\Longrightarrow$ Defining a time operator is tricky
- *Solution:* Only rely on 'symmetric + densely defined' and **positive operator-valued measures (POVMs)**
- POVMs avoid monotonicity no-go thm., too. [UW89; HSL21]
- *Example:* **Harmonic oscillator 'clock'** [BGL94; BGL95]
$$\hat{H}_{\mathrm{C}} = \hat{n}_{\mathrm{C}} + \frac{1}{2}\mathbb{1}_{\mathrm{C}}.$$

Define non-unitary $\hat{W}$ through
$$\hat{a} = \hat{W}\widehat{|a|}, \qquad \text{with} \qquad \widehat{|a|} := \hat{n}^{1/2}$$
having improper eigenstates $|\theta\rangle$
$$\hat{W}\,|\theta\rangle = e^{i\theta}\,|\theta\rangle\,, \qquad \text{with} \qquad |\theta\rangle = \sum_{n\geq 0} e^{in\theta}\,|n\rangle\,.$$
The relevant POVM:
$$B_0(X) := \frac{1}{2\pi}\int_X \mathrm{d}\theta\,|\theta\rangle\,\langle\theta| = \sum_{n,m\geq 0}\frac{1}{2\pi}\int_X e^{i(n-m)\theta}\,\mathrm{d}\theta\,|n\rangle\,\langle m|\,.$$
Giving one of many possible time operators as:
$$\hat{T}_0 = B_0(\theta) = \sum_{n\neq m\geq 0}\frac{1}{i(n-m)}\,|n\rangle\,\langle m| + \pi\mathbb{1}.$$

Observational Entropy

- **Entropy in closed systems** is difficult to define
- A recent collection of proposals is **observational entropy** [ŠDA19], providing in particular entropy for systems with Hamiltonian
$$\hat{H} = \hat{H}^{(1)} + \cdots + \hat{H}^{(m)} + \epsilon\hat{H}_{\mathrm{int}}$$
on a Hilbert space
$$\mathcal{H} = \mathcal{H}^{(1)} \otimes \cdots \otimes \mathcal{H}^{(m)}:$$

1. Pick conserved quantities $\hat{A}_1, \ldots, \hat{A}_n$, spectral decompose them, form spectral projection operators $\hat{P}_i$
2. Define
$$V_{i_1,\ldots,i_n} := \mathrm{tr}\left(\hat{P}_{i_n}\ldots\hat{P}_{i_2}\hat{P}_{i_1}\hat{P}_{i_2}\ldots\hat{P}_{i_n}\right),$$
$$p_{i_1,\ldots,i_n} := \mathrm{tr}\left(\hat{P}_{i_n}\ldots\hat{P}_{i_1}\hat{\rho}\hat{P}_{i_1}\ldots\hat{P}_{i_n}\right)$$
3. Get **coarse-grained entropy** even for closed systems as
$$S_{O(\mathcal{C}_1,\ldots,\mathcal{C}_n)}(\hat{\rho}) = -\sum_{i_1,\ldots,i_n} p_{i_1,\ldots,i_n}\ln\left(\frac{p_{i_1,\ldots,i_n}}{V_{i_1,\ldots,i_n}}\right),$$
where $O(\mathcal{C}_1,\ldots,\mathcal{C}_n)$ is an ordered set of collections C_i of above projection operators.

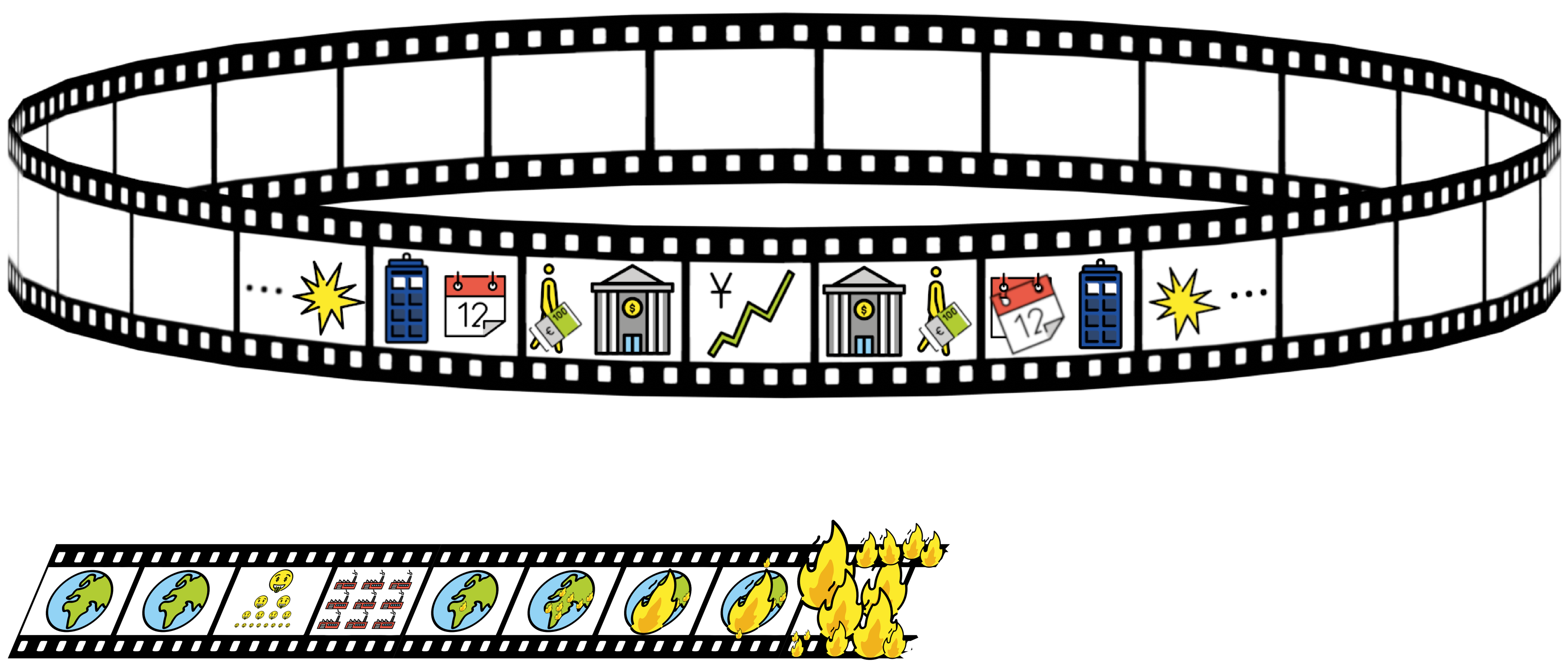
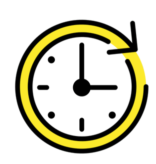
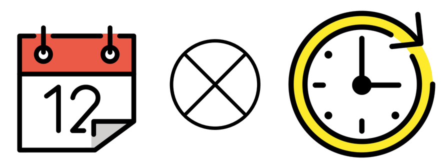


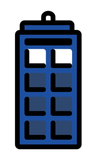
Figure: The realm of quantum gravity modelled as the space in which subsystems with internal, relational time exist. These subsystems are represented as reels of film, the quantum gravity substrate as the emptiness around it. Top: A subsystem corresponding to relational time with (potential) time-travel. Bottom: A subsystem with relational time and an interaction with the surrounding quantum gravity systems, demonstrating how a subsystem's notion of time might be lost in the overall system.



(a) A periodic clock.



(b) A periodic clock supplemented by a calendar.



(c) Actual time-travel, self-consistent or not.

Figure: The three options to physically distinguish in a model.

Toy Models & Inspiration

Goals

- Distinguish the following cases of systems with relational dynamics:
  1. System with a periodic clock
  2. System with a periodic clock and a memory/calendar
  3. System undergoing time travel (without Novikov self-consistency condition)
  4. System undergoing time travel (with Novikov self-consistency condition)
- *Can* we even distinguish cases 2 and 3?
- For given toy models, can we find **entropic reasons for disfavouring time travel** even in absence of a fundamental notion of time?
- Can this be related to earlier toy models of quantum gravity?

A First Model

- Minisuperspace model of a **closed Friedmann universe** with **conformally coupled scalar** χ:
$$\hat{H}\Psi(a,\chi) = \left(\frac{\partial^2}{\partial a^2} - \omega_a^2 a^2 - \frac{\partial^2}{\partial\chi^2} + \omega_\chi^2\chi^2\right)\Psi = 0$$
- Normalizability of Ψ gives two integers n_a, n_χ fulfilling
$$\frac{\omega_a}{\omega_\chi} = \frac{2n_\chi + 1}{2n_a + 1}$$
- Here, time is sometimes identified with the value of a
- Concrete questions:
  - What happens if we perform the **PW formalism** for the cyclic time operator given through the **POVM methods** above?
  - Does this give a first adequate toy model of either **time travel or a periodic clock?**
  - Can we **contrast or connect** this with other proposals for emergent **notions of time in canonical quantum gravity?**

Possible Extensions

- Subdivide the Hilbert space up to 4-partite:
$$\mathcal{H} = \mathcal{H}_{\mathrm{env}} \otimes \mathcal{H}_{\mathrm{C}} \otimes \mathcal{H}_{\mathrm{Q}} \otimes \mathcal{H}_{\mathrm{M}}$$
where
  - $\mathcal{H}_{\mathrm{env}}$: Surrounding/remaining, **quantum gravity environment**
  - $\mathcal{H}_{\mathrm{C}}$: **Clock**
  - $\mathcal{H}_{\mathrm{Q}}$: Quantum **system** with relational time
  - $\mathcal{H}_{\mathrm{M}}$: Memory or **calendar**
- Add **interactions**; *e.g.*, can we adapt a Jayne–Cummings model to the Wheeler–DeWitt equation?

Acknowledgements

Sebastian Schuster acknowledges support from the technical and administrative staff at the Charles University, and financial support from Czech Science Foundation grant GACR 23-07457S.
The figures were created using characters U+1F525, U+1F30D, U+1F4C6, U+1F4C5, U+1F911, U+1F4B6, U+1F4B9, U+1F4A5, U+1F6B6, U+1F3ED, U+1F3E6, U+E0AB extra emoji E149 of the OpenMoji library (https://openmoji.org/library/) published under Creative Commons license CC BY-SA 4.0, and https://freesvg.org/filmstrip-vector-image published into the public domain.

References

BGL94 P. Busch, M. Grabowski & P. J. Lahti. Time observables in quantum theory. *Physics Letters A* **191,** 357–361. doi:10.1016/0375-9601(94)90785-4 (Aug. 1994).
BGL95 P. Busch, M. Grabowski & P. J. Lahti. Who Is Afraid of POV Measures? Unified Approach to Quantum Phase Observables. *Annals of Physics* **237,** 1–11. doi:10.1006/aphy.1995.1001 (Jan. 1995).
Fri+90 J. Friedman *et al.* Cauchy problem in spacetimes with closed timelike curves. *Physical Review D* **42,** 1915–1930. doi:10.1103/PhysRevD.42.1915 (Sept. 1990).
GLM15 V. Giovannetti, S. Lloyd & L. Maccone. Quantum Time. *Physical Review D* **92,** 045033. doi:10.1103/PhysRevD.92.045033. arXiv: 1504.04215 [quant-ph] (Aug. 2015).
HSL21 P. A. Höhn, A. R. H. Smith & M. P. E. Lock. The Trinity of Relational Quantum Dynamics. *Physical Review D* **104,** 066001. doi:10.1103/PhysRevD.104.066001. arXiv: 1912.00033 [quant-ph] (Sept. 2021).
Kie12 C. Kiefer. *Quantum Gravity.* 3rd ed. *International Series of Monographs on Physics* **136.** ISBN: 978-0-19-958520-5 (Oxford University Press, 2012).
MV17 C. Marletto & V. Vedral. Evolution without evolution, and without ambiguities. *Physical Review D* **95,** 043510. doi:10.1103/PhysRevD.95.043510. arXiv: 1610.04773 [quant-ph] (Feb. 2017).
PW83 D. N. Page & W. K. Wootters. Evolution without evolution: Dynamics described by stationary observables. *Physical Review D* **27,** 2885–2892. doi:10.1103/PhysRevD.27.2885 (June 1983).
ŠDA19 D. Šafránek, J. M. Deutsch & A. Aguirre. Quantum coarse-grained entropy and thermalization in closed systems. *Physical Review A* **99,** 012103. doi:10.1103/PhysRevA.99.012103. arXiv: 1803.00665 [quant-ph] (Jan. 2019).
UW89 W. G. Unruh & R. M. Wald. Time and the interpretation of canonical quantum gravity. *Physical Review D* **40,** 2598–2614. doi:10.1103/PhysRevD.40.2598 (Oct. 1989).