Radially infalling detector in BTZ spacetime

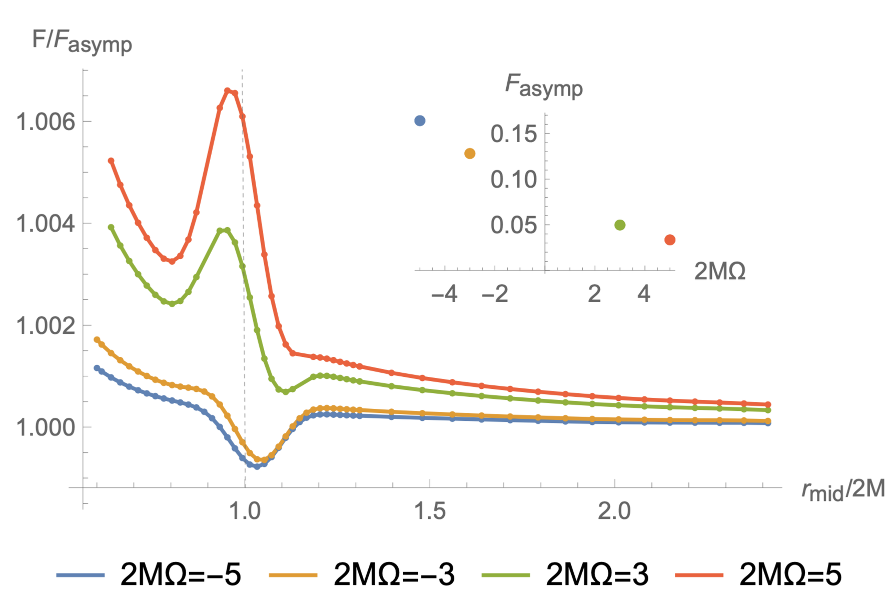
María R. Preciado-Rivas [1,2] Manar Naeem [3,2] Robert B. Mann [3,2,4] Jorma Louko [5]

[1]Department of Applied Mathematics, University of Waterloo [2]Institute for Quantum Computing, University of Waterloo [3]Department of Physics and Astronomy, University of Waterloo [4]Perimeter Institute for Theoretical Physics [5]School of Mathematical Sciences, University of Nottingham

Introduction

***A little excitement across the horizon* (2022)**

- Ng *et al.* numerically calculated the transition probability of an Unruh-DeWitt detector that is radially infalling a four-dimensional Schwarzschild black hole.
- The transition probability, as a function of the midpoint of the interaction interval, attains an unexpected local extremum near the event horizon.



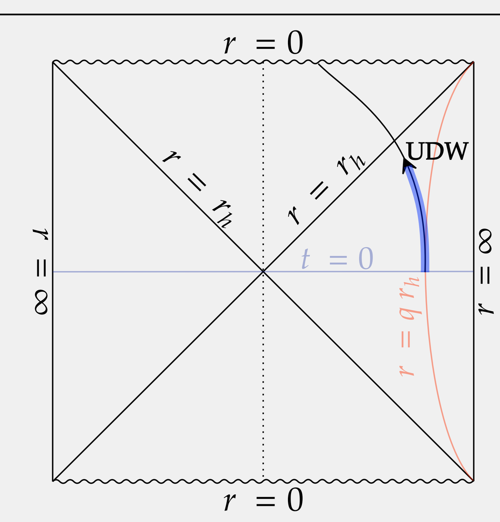
Is there "a little excitement across the horizon" in other spacetimes?

- In 2012, Hodgkinson and Louko numerically calculated the transition rate of an UDW detector that is radially infalling a three-dimensional BTZ black hole.
- However, they did not calculate the transition rate across or near the event horizon.

Setup

- Following the work by Hodgkinson and Louko, we consider a radially infalling timelike geodesic in a spinless BTZ black hole spacetime:

$$t = \left(\ell/\sqrt{M}\right) \operatorname{arctanh}\left(\frac{\tan\tilde{\tau}}{\sqrt{q^2-1}}\right),$$
$$r = \ell\sqrt{M} q \cos\tilde{\tau},$$
$$\phi = \phi_0.$$



- M is the mass of the black hole, $q > 1$ determines the initial position of the detector, $\tilde{\tau}$ is an affine parameter such that the detector's proper time equals $\tau = \tilde{\tau}\ell$, and ℓ is the radius of AdS_3 space.

- The UDW detector is an idealized pointlike two-level quantum system, with energy levels 0 and E. It interacts with a massless conformally coupled scalar field in the Hartle-Hawking vacuum.
- The transition rate of a detector, that is switched on at $\tau_0 = \tau - \Delta\tau$ and switched off at τ, is given by

$$\dot{\mathcal{F}}_\tau(E) = \frac{1}{4} + 2\int_0^{\Delta\tau} \mathrm{d}s\, \mathrm{Re}\left[\mathrm{e}^{-iEs} W_0(\tau, \tau - s)\right]$$

where $W_0(\tau, \tau - s)$ is the pullback of the scalar field's Wightman function to the detector's worldline.

- The Wightman function on BTZ spacetime is expressed as an image sum of the AdS_3 Wightman function: $G_{\mathrm{BTZ}}(\mathsf{x}, \mathsf{x}') = \sum_n G_{\mathrm{A}}(\mathsf{x}, \Lambda^n \mathsf{x}')$, where $\Lambda\mathsf{x}'$ denotes the action on x' of the group element $(t, r, \phi) \to (t, r, \phi + 2\pi)$.
- The final expression for the transition rate is

$$\dot{\mathcal{F}}_\tau(E) = 1/4 + \frac{1}{2\pi\sqrt{2}} \sum_{n=-\infty}^{\infty} \int_0^{\Delta\tilde{\tau}} \mathrm{d}\tilde{s}\, \mathrm{Re}\left[\frac{\mathrm{e}^{-i\tilde{E}\tilde{s}}}{\sqrt{-1 + f_n(\tilde{\tau}, \tilde{s})}} - \zeta \frac{\mathrm{e}^{-i\tilde{E}\tilde{s}}}{\sqrt{1 + f_n(\tilde{\tau}, \tilde{s})}}\right],$$

where

$$f_n(\tilde{\tau}, \tilde{s}) := K_n \cos\tilde{\tau}\cos(\tilde{\tau} - \tilde{s}) + \sin\tilde{\tau}\sin(\tilde{\tau} - \tilde{s}) \quad \text{and}$$
$$K_n := 1 + 2q^2 \sinh^2\left(n\pi\sqrt{M}\right),$$

with $\Delta\tilde{\tau} := \Delta\tau/\ell$ and $\tilde{E} := E\ell$. $\zeta \in \{-1, 0, 1\}$ specifies the boundary condition at infinity for the Wightman function: Neumann, transparent, or Dirichlet, respectively.

- The term $n = 0$ gives the transition rate of a detector on a geodesic in pure AdS_3.
- The argument of the square root in the first term of the integral can change of sign within the integration interval for $n \geq 1$. As a result, the transition rate is non-smooth at $\Delta\tilde{\tau}_n = \arccos(1/K_n)$.

Results

- The transition rate defers considerably from the zeroth term $\dot{\mathcal{F}}_\tau^{n=0}$ for detection times near and beyond $\Delta\tau_1/\ell$, with the difference being more pronounced as more glitches $\Delta\tau_n/\ell$ are reached.

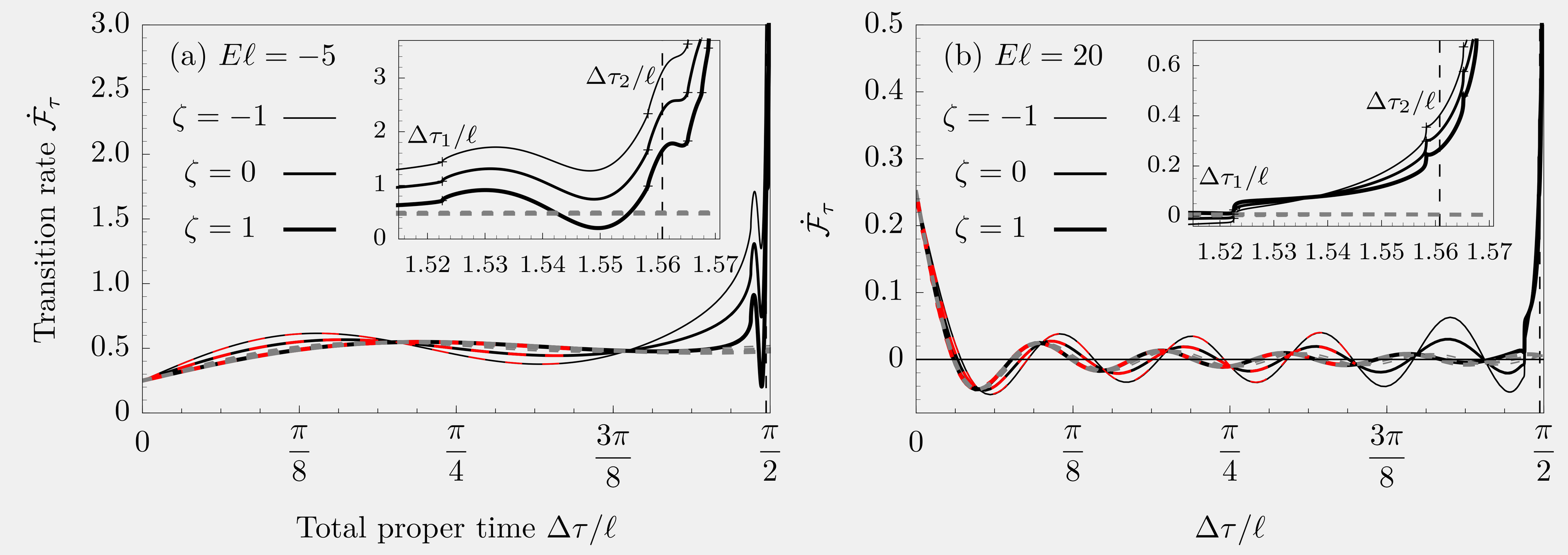


Figure 1. $M = 10^{-4}$, $q = 100$, $\tau_0 = 0$. The event horizon of the BTZ black hole is indicated with a dashed vertical line at $\Delta\tau/\ell = \arccos(1/q)$.

- The deviation from the zeroth term is determined by the glitches $\Delta\tau_n/\ell$, which can be tweaked varying M and q.

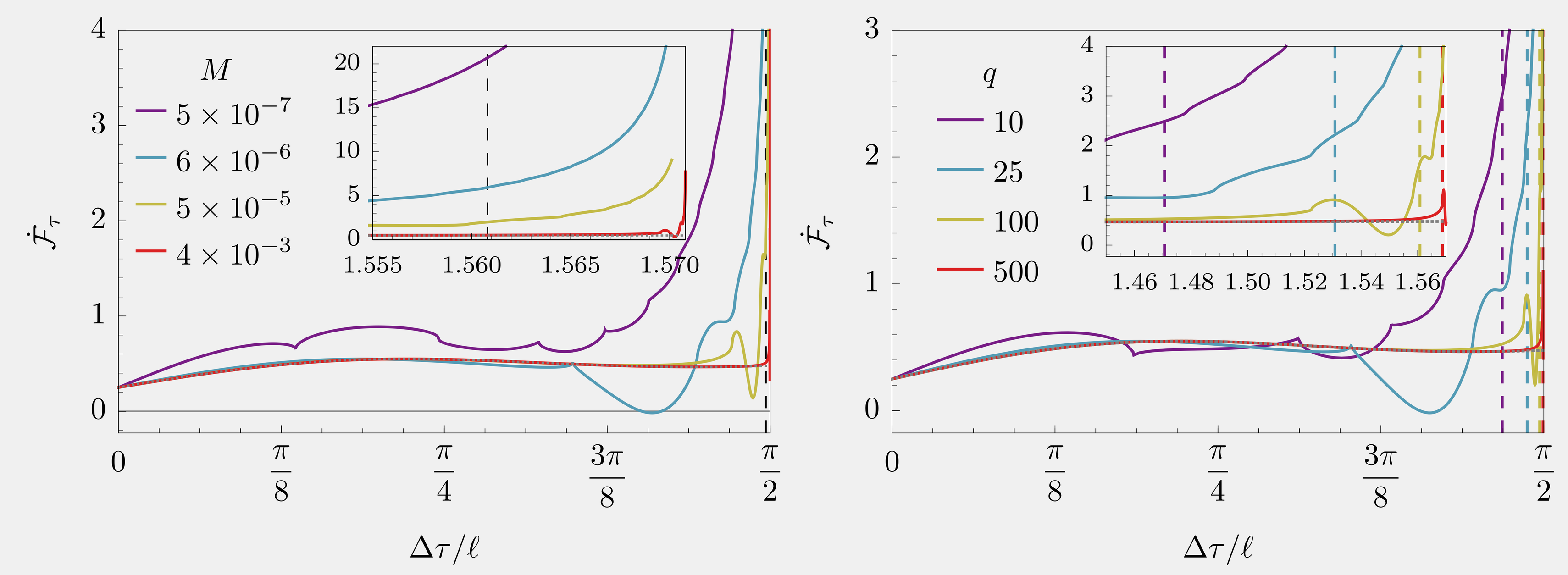


Figure 2. $E\ell = -5$, $\tau_0 = 0$, $q = 100$ (left) or $M = 10^{-4}$ (right).

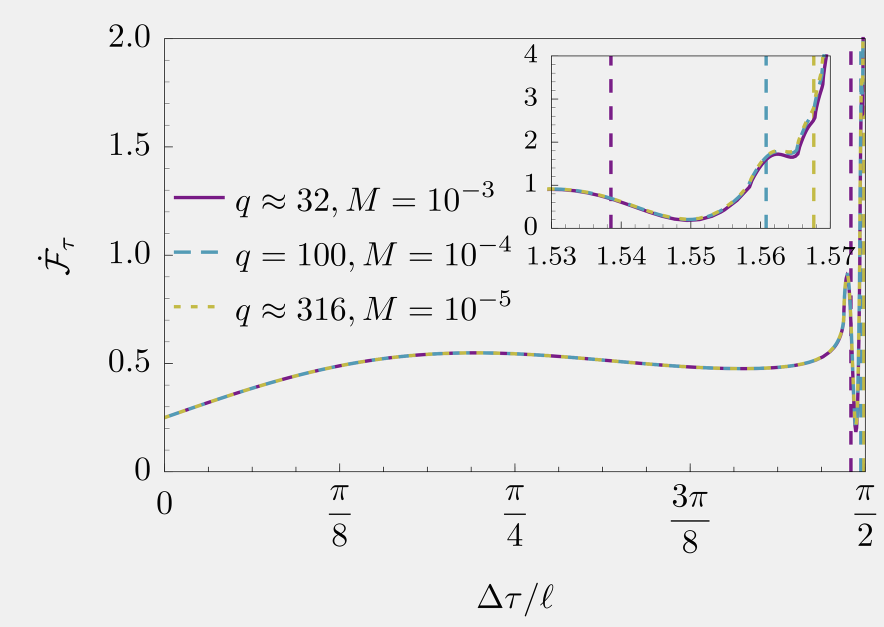


Figure 3. $E\ell = -5$, $\tau_0 = 0$

Discussion

- We extended the work by Hodgkinson and Louko for the transition rate near and across the event horizon. We found that it increases rapidly as the singularity is approached. $\Delta\tau/\ell = \pi/2$ is where the identifications break down, and the image sum does not converge.
- For certain parameters, we observe a local extremum as the detector crosses the event horizon. This feature in the transition rate for a free falling detector in a BTZ black hole resembles the local extremum that was observed in the work by Ng et al. for the response function of a detector that is free falling into a Schwarzschild black hole.
- However, the black hole mass and the detector's initial radial position can be varied such that the same features appear closer of farther from the horizon crossing.
- Future work will focus on calculating the response function for the BTZ case so that a direct comparison can be made between the Schwarzschild and BTZ black holes.

References

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mrprecia@uwaterloo.ca