Entanglement-Assisted Effective Models for Tests of Fundamental Physics in Atom Interferometry

BLUESKY PROJECT
MuMo-RmQM

Institute of Quantum Physics

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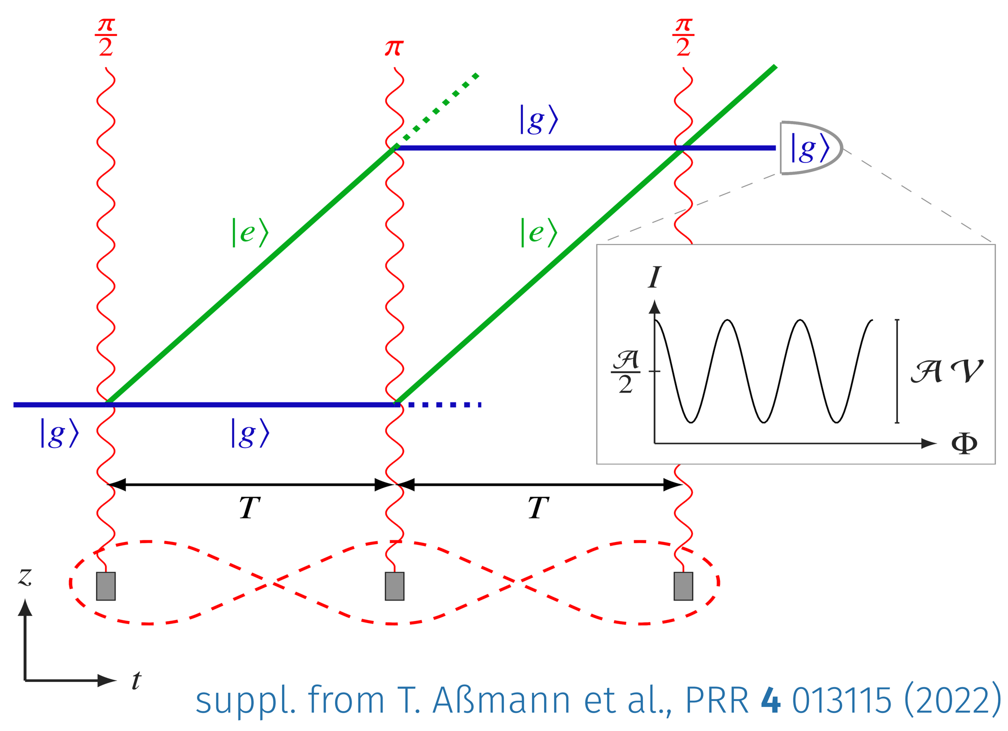
Motivation - Why is a full field theory for matter-wave optics/quantum metrology interesting?



suppl. from T. Aßmann et al., PRR **4** 013115 (2022)

Leveraging quantum entanglement in atom interferometry

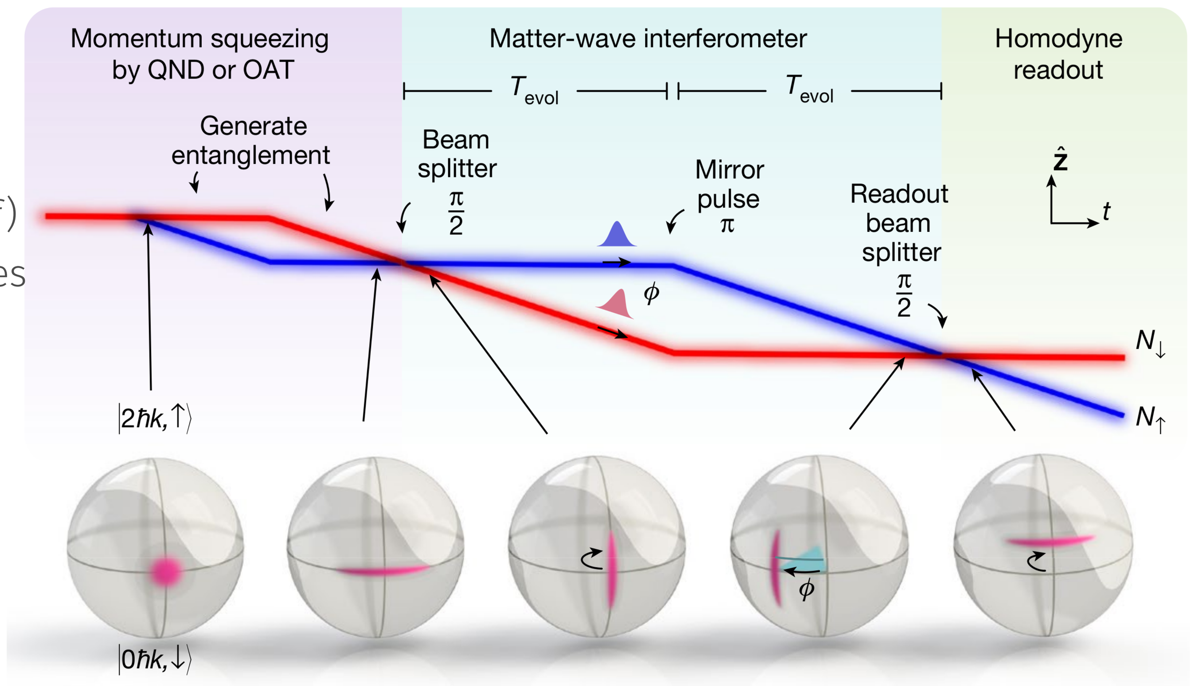
Quantum phase estimation beyond Standard Quantum Limit (SQL) via cavity Raman experiments [8]

→ Generation of entanglement between external degrees of freedom (dof)

→ Our theoretical effective model can describe 2-photon Raman processes in a cavity using effective two-mode field theory

Theory with capability to describe entanglement between motional and internal dofs

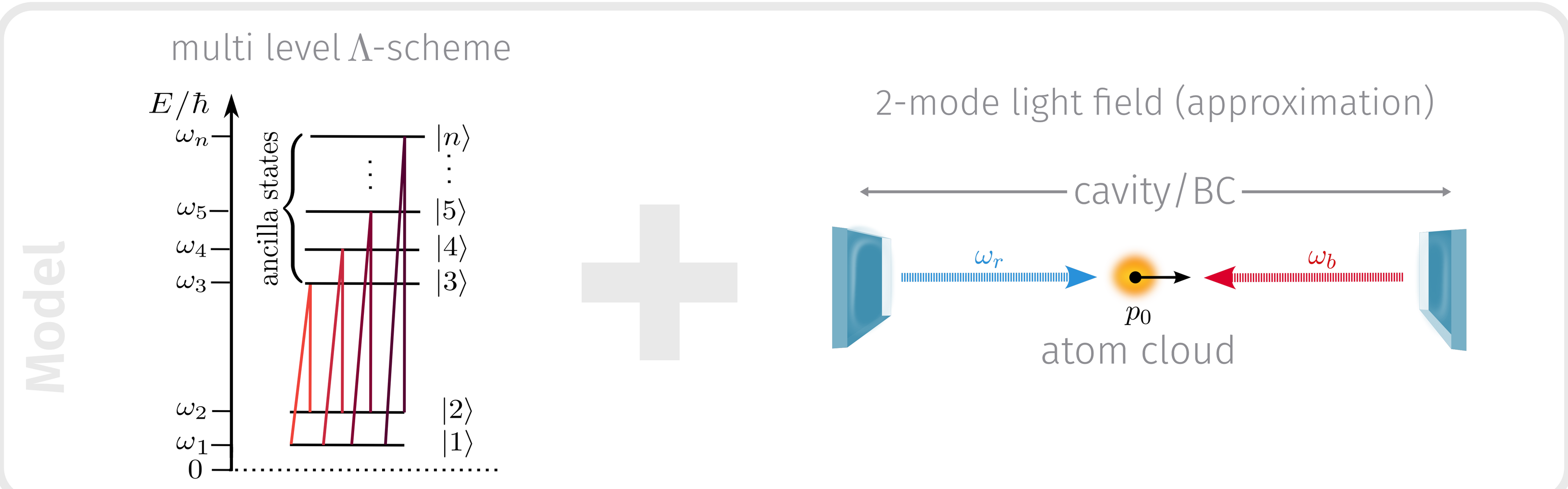
Future: Entanglement dynamics of beam-splitters



suppl. from [8]: G. P. Greve et al., Nature **610**, 472–477 (2022)

Results/Example - Modelling two-photon processes in a cavity

1. Quantum matter-wave optics setup



Atom

$$\hat{H}_{\mathrm{A}} = \int \mathrm{d}^3R \sum_{\ell=1}^{n} \hat{\psi}_\ell^\dagger(\boldsymbol{R}) \left(\hat{\mathcal{H}}_{\mathrm{COM}}^{(\ell)} + \hbar\omega_\ell\right) \hat{\psi}_\ell(\boldsymbol{R})$$

Light

$$\hat{H}_{\mathrm{L}} = \sum_{\alpha=a,b} \hbar\omega_\alpha \left(\hat{\alpha}^\dagger\hat{\alpha} + \tfrac{1}{2}\right)$$

System Hamiltonian $\hat{H} = \hat{H}_{\mathrm{A}} + \hat{H}_{\mathrm{AL}} + \hat{H}_{\mathrm{L}}$

Interaction

$$\hat{H}_{\mathrm{AL}} = \int \mathrm{d}^3R \sum_{j=3}^{n} \Big[\hat{\Psi}_j^\dagger(\boldsymbol{R})\mathcal{G}_{j2}(\boldsymbol{R})(\hat{b}+\hat{b}^\dagger)\hat{\Psi}_2(\boldsymbol{R}) + \hat{\Psi}_j^\dagger(\boldsymbol{R})\mathcal{G}_{j1}(\boldsymbol{R})(\hat{a}+\hat{a}^\dagger)\hat{\Psi}_1(\boldsymbol{R})\Big] + \mathrm{h.c.}$$

System partitioning

$$i\hbar\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\hat{\Psi}_n(\boldsymbol{R})\\ \vdots \\ \hat{\Psi}_3(\boldsymbol{R}) \\ \hat{\Psi}_2(\boldsymbol{R}) \\ \hat{\Psi}_1(\boldsymbol{R})\end{pmatrix} = \hbar\begin{pmatrix}\Delta_n & 0 & 0 & \hat{\Omega}_{n2} & \hat{\Omega}_{n1}\\ 0 & \ddots & 0 & \vdots & \vdots\\ 0 & 0 & \Delta_3 & \hat{\Omega}_{32} & \hat{\Omega}_{31}\\ \hat{\Omega}_{n2}^\dagger & \dots & \hat{\Omega}_{32}^\dagger & \Delta_2 & 0\\ \hat{\Omega}_{n1}^\dagger & \dots & \hat{\Omega}_{31}^\dagger & 0 & \Delta_1\end{pmatrix}\begin{pmatrix}\hat{\Psi}_n(\boldsymbol{R})\\ \vdots \\ \hat{\Psi}_3(\boldsymbol{R}) \\ \hat{\Psi}_2(\boldsymbol{R}) \\ \hat{\Psi}_1(\boldsymbol{R})\end{pmatrix}$$

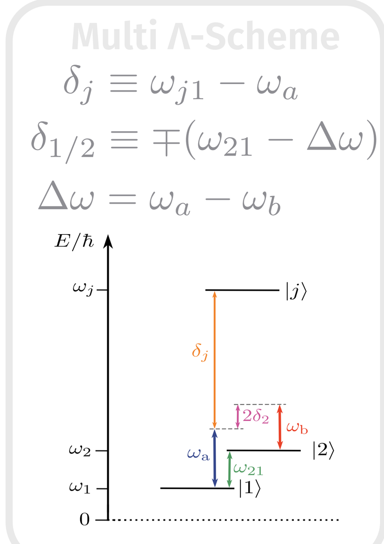
irrelevant dof

relevant dof

2. Two-photon Rabi operators via adiabatic elimination

Starting point: Matter field-operator Heisenberg equation of motion

$$i\hbar\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\hat{\Psi}_a(\boldsymbol{R})\\ \hat{\psi}(\boldsymbol{R})\end{pmatrix} = \hbar\begin{pmatrix}\hat{\Delta}_{3n} & \hat{\Omega}\\ \hat{\Omega}^\dagger & \hat{\Delta}_{12}\end{pmatrix}\begin{pmatrix}\hat{\Psi}_a(\boldsymbol{R})\\ \hat{\psi}(\boldsymbol{R})\end{pmatrix}$$



$\delta_j \equiv \omega_{j1} - \omega_a$

$\delta_{1/2} \equiv \mp(\omega_{21} - \Delta\omega)$

$\Delta\omega = \omega_a - \omega_b$

Method: Adiabatic elimination via generalized projectors/resolvents [4,5]

Ansatz: $\hat{\Psi}_a(\boldsymbol{R}) = \mathcal{P}(t)\hat{\Psi}(\boldsymbol{R})$

Result:

$$\hat{H}_{2\mathcal{P}} \approx \int \mathrm{d}^3R\Big(\hat{\Delta}_{12}(\boldsymbol{R},-i\hbar\nabla) - \hbar\sum_{j=3}^{n}\hat{\Omega}^\dagger(\boldsymbol{R})\hat{\psi}_j^\dagger(\boldsymbol{R})\hat{\psi}_j(\boldsymbol{R})\delta_j^{-1}\hat{\Omega}(\boldsymbol{R}) + \mathcal{O}(\{\delta_j^{-2}\})\Big)$$

Example: Single particle like Rabi Hamiltonian for Λ-system

$$\hat{H}_{2\mathcal{P}} \approx \int \mathrm{d}^3R\Big(\hat{\Delta}_{12}(\boldsymbol{R},-i\hbar\nabla) - \hat{\Omega}^\dagger(\boldsymbol{R})\hat{\psi}_3^\dagger(\boldsymbol{R})\hat{\psi}_3(\boldsymbol{R})\delta_3^{-1}\hat{\Omega}(\boldsymbol{R})\Big)$$

$$\hat{\Delta}_{12} = \sum_{j=1}^{2}\hat{\psi}_j^\dagger(\boldsymbol{R})\hbar\hat{\Delta}_j(\boldsymbol{R},-i\hbar\nabla_{\boldsymbol{R}})\hat{\psi}_j(\boldsymbol{R})$$

$$\hbar\hat{\Delta}_{1/2}(\boldsymbol{R}) \equiv \mathcal{H}_{\mathrm{COM}}^{(1/2)}(\boldsymbol{R}) + \hbar\delta_{1/2} + \hbar\omega_{a/b}$$

$$-\frac{\hbar}{\delta_3}\Big[\sum_{m=1}^{2}\hat{\Omega}_{3m}^\dagger\hat{\psi}_3^\dagger(\boldsymbol{R})\hat{\psi}_3(\boldsymbol{R})\hat{\Omega}_{3m} + \left(\hat{\Omega}_{31}^\dagger\hat{\psi}_1^\dagger(\boldsymbol{R})\hat{\psi}_2(\boldsymbol{R})\hat{\Omega}_{32} + \mathrm{h.c.}\right)\Big]$$

"AC-Stark+↺"

Averaging "Bloch-Siegert shift & two-photon light shifts"

$$-\frac{\hbar}{\delta_3}\sum_{j=1}^{2}\hat{\psi}_j^\dagger(\boldsymbol{R})|\mathcal{G}_{3j}(\boldsymbol{R})|^2\left(2\hat{N}_\alpha + \mathbb{1}_\alpha + (e^{-i\omega_\alpha t}\hat{\alpha}^2 + \mathrm{h.c.})\right)\hat{\psi}_j(\boldsymbol{R})$$

"two-photon transition+↺"

$$\hat{\psi}_1^\dagger(\boldsymbol{R})\mathcal{G}_{31}^*(\boldsymbol{R})\left((e^{i\Delta\omega t}\hat{a}^\dagger\hat{b} + \mathrm{h.c.}) + e^{-i\omega_a t}\hat{a}\hat{b} + e^{i\omega_b t}\hat{a}^\dagger\hat{b}^\dagger\right)\mathcal{G}_{32}(\boldsymbol{R})\hat{\psi}_2(\boldsymbol{R})$$

Outlook: Getting rid of residual time-dependencies via average Hamiltonian theory [7]!

$$\hat{H} = \hat{H}_0 + \sum_n\left(\hat{f}_n e^{-i\omega_n t} + \mathrm{h.c.}\right) \qquad \hat{H}_{\mathrm{eff}} = \hat{H}_0 + \sum_{n,m}\frac{[\hat{h}_m,\hat{h}_n]}{\hbar\omega_{nm}^{(+)}}e^{i(\omega_m-\omega_n)t}$$

$$\mathcal{L}_{\mathrm{eff}} \cong \frac{1}{\hbar}\left(i[\hat{H}_{\mathrm{eff}},\cdot] - \mathcal{D}[\cdot]\right)$$

$$\hat{O}(t) = \mathcal{T}\exp\left(i\int_{t_0}^{t}\mathrm{d}s\,\mathcal{L}(s)\right)\hat{O}(t_0) \qquad \mathcal{D}[\cdot] = \mathcal{D}_L\left[\cdot,\{\hat{L}_m = \hat{f}_m e^{-i\omega_m t}\},\{\hbar\omega_{mn}^{(-)}\}\right]$$

3. Two-mode two-photon Rabi model in RWA for short pulses

Simplest case: Rotating-wave approx. in Hamilton density and insert mode functions of plane waves

$$\hat{H}_{2\mathcal{P}} = \int \mathrm{d}^3R\left(\hat{\psi}_2^\dagger(\boldsymbol{R}),\hat{\psi}_1^\dagger(\boldsymbol{R})\right)\hat{h}_{2\mathcal{P}}^{(\mathrm{SP})}(\boldsymbol{R},-i\hbar\nabla_{\boldsymbol{R}})\begin{pmatrix}\hat{\psi}_2(\boldsymbol{R})\\ \hat{\psi}_1(\boldsymbol{R})\end{pmatrix}$$

$$\hat{h}_{2\mathcal{P}}^{(\mathrm{SP})} = \begin{pmatrix}\hat{h}_{22}(\boldsymbol{R},-i\hbar\nabla_{\boldsymbol{R}}) & \hat{h}_{21}(\boldsymbol{R})\\ \hat{h}_{12}(\boldsymbol{R}) & \hat{h}_{11}(\boldsymbol{R},-i\hbar\nabla_{\boldsymbol{R}})\end{pmatrix}$$

$$\mathcal{G}_{3j}(\boldsymbol{R}) \equiv \frac{E_{0,\alpha}}{\hbar}\boldsymbol{d}_{3j}\boldsymbol{u}_\alpha = \Omega_{3j,\alpha}e^{i(\boldsymbol{k}_\alpha\cdot\boldsymbol{r}+\Phi_\alpha)}$$

Rabi frequency $\Omega \equiv \frac{\Omega_{31,a}^*\Omega_{32,b}}{\delta_3}$

Adiabatic chirp $\theta \equiv \Delta\boldsymbol{k}\cdot\boldsymbol{r} + \Delta\Phi + \phi(t) - \Delta\omega t$, $\Delta\Phi \equiv \Phi_a - \Phi_b$

Two-photon coupling $\hat{h}_{21} \equiv -\frac{\hbar|\Omega|}{2}e^{-i\theta(\boldsymbol{R})}\hat{a}^\dagger\hat{b}$

SU(2)-like decomposition of effective Hamiltonian

$$\hat{J}_+ = \int \mathrm{d}^3R\,\hat{\Psi}_2^\dagger(\boldsymbol{R})\hat{h}_{21}\hat{\Psi}_1(\boldsymbol{R}) = \hat{J}_-^\dagger$$

$$\hat{J}_{0/3} = \tfrac{1}{2}\int \mathrm{d}^3R\left(\hat{\Psi}_2^\dagger(\boldsymbol{R})\hat{h}_{22}\hat{\Psi}_2(\boldsymbol{R}) \pm \hat{\Psi}_1^\dagger(\boldsymbol{R})\hat{h}_{11}\hat{\Psi}_1(\boldsymbol{R})\right)$$

$$\hat{H}_{2\mathcal{P}} = \hat{J}_0 + \hat{J}_+ + \hat{J}_- + \hat{J}_3$$

Common quantum Stark shifts + CM-motion; Two-photon transition; Differential quantum Stark shifts + CM-motion

Approximate Time-evolution for δ-switching [5, 7]

Pulse area $\mathcal{A} = \Gamma\tau$

$$\hat{H}_{2\mathcal{P}} \mapsto \hat{H}_{2\mathcal{P}}(t;t') \equiv \frac{\mathcal{A}}{\Gamma}\delta(t-t')\left(\hat{J}_0(t) + \hat{J}_+(t) + \hat{J}_-(t) + \hat{J}_3(t)\right), \quad t' \in (t_0,t), \quad \hat{J}_+(t) + \hat{J}_-(t) + \hat{J}_3(t) \equiv \hat{\mathcal{V}}(t)$$

$$\hat{U}(t,t_0) = \hat{U}_0(t,t_0)\hat{U}_{\mathrm{int}}(t,t_0) \simeq \exp\left\{-\frac{i}{\hbar}\mathcal{A}\frac{\hat{J}_0(t')}{\Gamma}\right\}\exp\left\{-\frac{i}{\hbar}\mathcal{A}\frac{\hat{\mathcal{V}}^{(0)}(t')}{\Gamma}\right\}$$

Common quantum Stark shifts + CM-motion

Heisenberg operators w.r.t. $\hat{J}_0(t')$

References and related work

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Acknowledgements & Contact

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