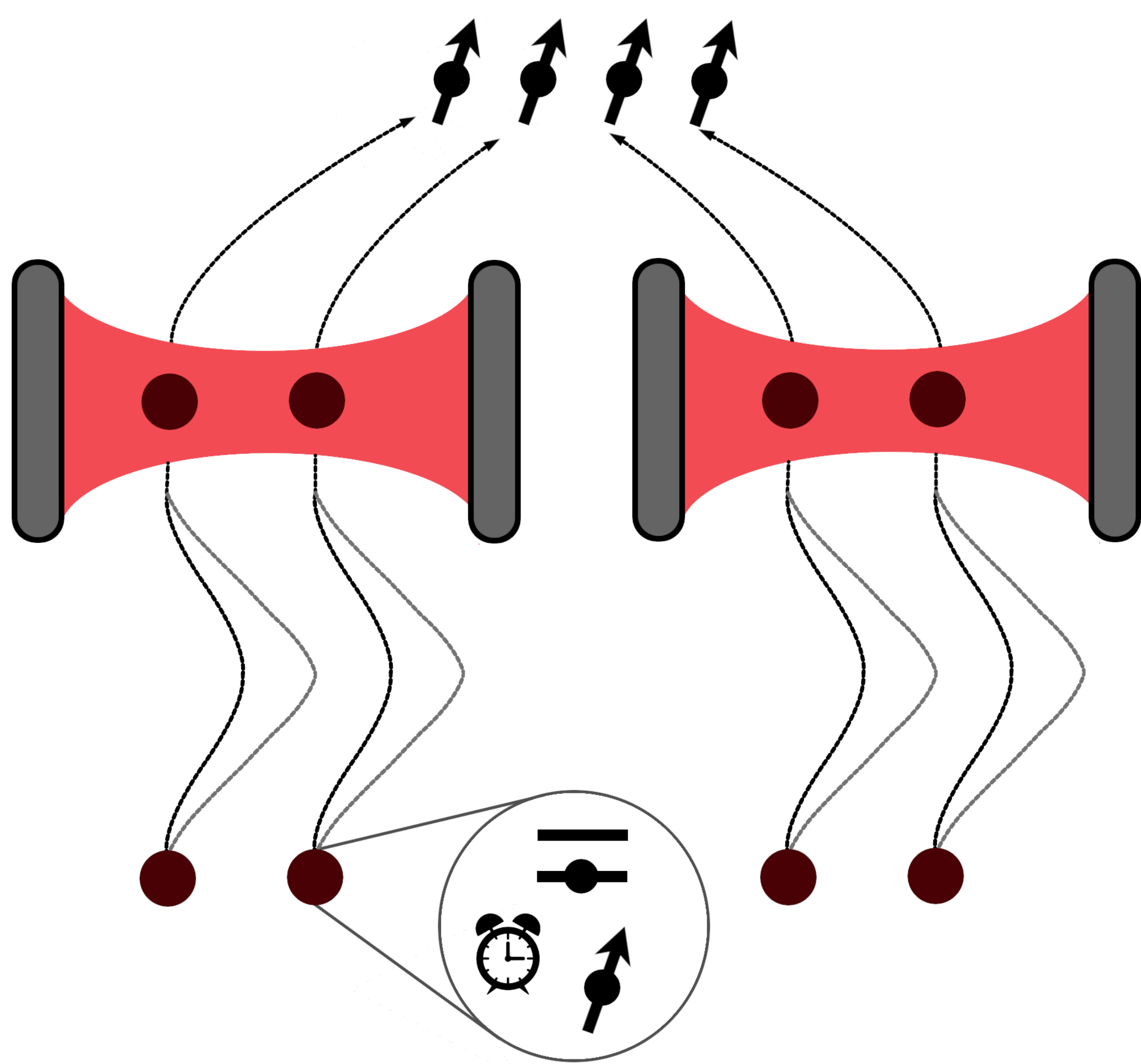


## Motivation & Research Goals

According to the general theory of relativity, time can flow at different rates depending on the configuration of massive objects, affecting the temporal order of events. Combined with quantum theory, this gravitational effect can result in events with an indefinite temporal order when a massive object is prepared in a suitable quantum state. This was argued to lead to a theory-independent test of the non-classical order of events through the violation of Bell-type inequalities for temporal order [2]. Here we show that the theory-independence of this protocol is problematic: one of the auxiliary assumptions in the above approach turns out to be essential, while it is explicitly theory-dependent [1]. Due to the Equivalence Principle, the same problem arises when one considers the gravitational case, and thus theory-dependent additional assumptions are needed behind Bell inequalities for temporal order to interpret a violation of the final inequalities as a signature of indefinite temporal order.

## Methods



Instead of using a massive object to control the spacetime geometry—and thus temporal order via gravitational time dilation—here we want to control the trajectories of particles so as to induce special-relativistic time dilation.

The protocol involves several degrees of freedom, which can be thought of as particles ‘glued’ together:

- The clock is a particle with some time-evolving internal state. The role of the clock is to trigger an interaction between the detector and the cavity at the desired proper time.
- The detector is a particle with two internal energy levels that interacts with a quantum field confined in a cavity. The detector interacts with the cavity via the Unruh-DeWitt Hamiltonian:

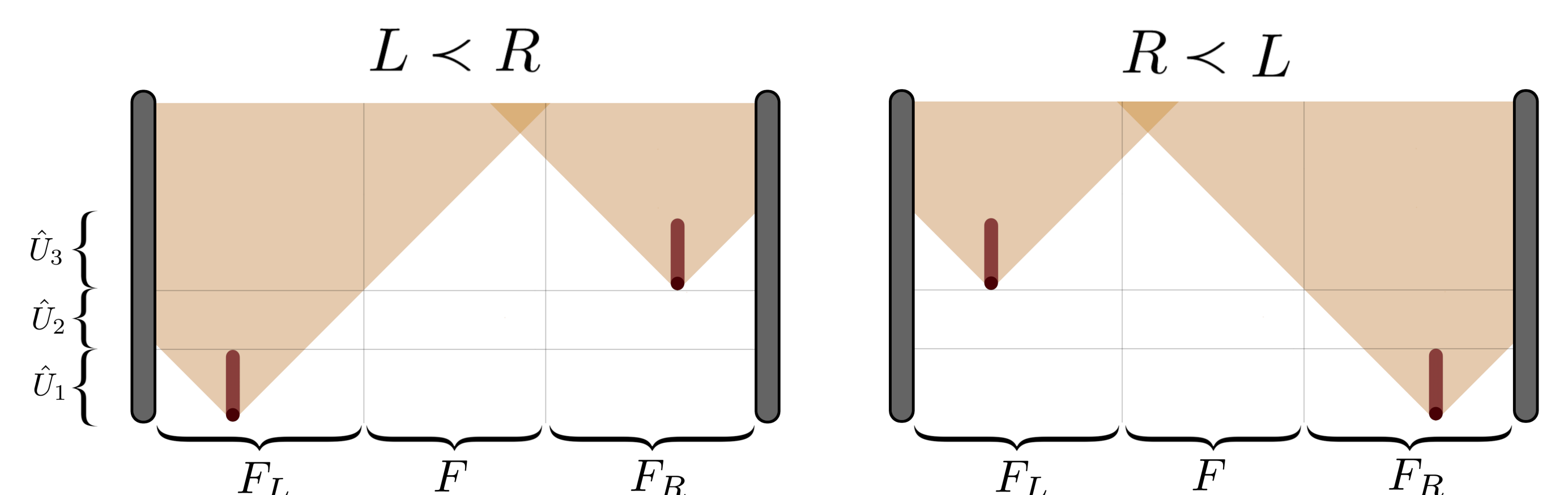
$$\hat{H}_{\text{UDW}} = \lambda \chi_d(t) \hat{\mu}_S \hat{\phi}(x_d),$$

where  $\lambda$  is a dimensionless coupling constant;  $\chi_d(t)$  is the switching function;  $\hat{\mu}_S$  is the monopole operator. Finally,  $\hat{\phi}(x_d)$  is the field operator evaluated at the position of the detector.

- The control is a spin- $\frac{1}{2}$  particle, whose two orthogonal spin states serve to define the molecules’ trajectories.

## Selected Results


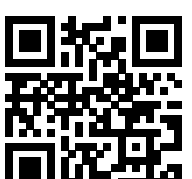
- We constructed a non-gravitational scenario where accelerating particles, interacting with quantum fields, according to their own internal clock degrees of freedom, can lead to a violation of the temporal Bell inequalities analogous to the gravitational case.
- We described the procedure that would lead us to the violation of Bell’s inequalities for the proposed system that occurs also when the events responsible for the entanglement are space-like which we interpreted as an ambiguity in the signature of indefinite temporal order.
- We finally found that this surprising conclusion is the result of the failure of the additional assumption, that target systems have no other evolution except the one governed by unitaries applied in a specific time order.



**Spacetime diagram of a cavity interacting with two detectors.** Operators  $\hat{U}_1$  and  $\hat{U}_3$  describes the evolution of the system according to the interaction, and  $\hat{U}_2$  is an operator of the free evolution that occurs between interactions.  $F_L$  and  $F_R$  are parts of the cavity that can interact with the detector in some finite time.  $F$  is a middle segment of the cavity that evolves only due to the free evolution operator between interactions.

- We argued that in order to satisfy all assumptions of the Bell theorem for time order – including the auxiliary one (of no free evolution of the targets) – it is essential to invoke theory-dependent arguments. What is more, in a generic implementation – including a gravitational version of the protocol – this assumption is not met. Our chosen model thus serves as a means to clarify the overlooked aspect of Bell inequalities for temporal order. Consequently, our conclusions hold significance for any theoretical or experimental pursuit of indefinite temporal order.
- Our result opens the question of whether it is possible to formulate a stronger, theory-independent, test of temporal order. The insight from the present study is that it is problematic to separate out the effect of the free dynamics of the system from that of the local operations.

## References

- [1]  Indefinite temporal order without gravity  
K. Dębski, M. Zych, F. Costa and A. Dragan  
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- [2]  Bell’s theorem for temporal order  
M. Zych, F. Costa, I. Pikovski and Č. Brukner  
Nat. Commun. 10, 3772 (2019)