# Unruh Effect for SU(2) Qudit Detectors

**Everett Patterson**[1,2], Caroline Lima[1,2,3], Erickson Tjoa[1,2], and Robert B. Mann[1,2,3]

[1]Department of Physics and Astronomy, University of Waterloo, Canada; [2]Institute for Quantum Computing, University of Waterloo, Canada; [3]Perimeter Institute for Theoretical Physics, Waterloo, Ontario, Canada

## Motivation

- Thermality is an important feature of QFT
- UDW detectors (qubits) are frequently used to operationalize temperature observation
- But qubits often display special properties compared to general qudits
- Can detector thermalization be recovered using more general quantum detector models?

## Introduction

The Unruh effect is one of the most important predictions of quantum field theory (QFT). It stipulates that a constantly accelerating observer in the Minkowski vacuum actually perceives the presence of particles as if it were in a thermal bath with inverse temperature $\beta = a/(2\pi)$ proportional to the detectors proper acceleration. This temperature, $T_U = \beta^{-1}$ is known as the Unruh temperature.

This accelerated counterpart to Hawking's black hole radiation can be formulated in a few different ways:

1. Kubo-Martin-Schwinger (KMS) condition, describing the pullback of the Wightman function along a constantly accelerating trajectory as stationary and periodic in imaginary time.
2. Thermalization of Unruh-DeWitt (UDW) particle detector to a Gibbs state with temperature $T_U$.
3. In terms of the detailed balance condition, which states that the excitation-to-deexcitation ratio (EDR) is equal to $e^{-\beta\Omega}$, where $\Omega$ is the energy gap between two levels of a detector.

The detailed balance is often thought of as the smoking gun of an accelerating detector's thermalization.

Here we show that, in general, the detailed balance condition is an inadequate indicator of an accelerating qudit detector's thermalization. As such, it is not clear that the detailed balance condition is a necessary condition for thermalization in general.

## Set-up

Consider a massless scalar field in (3+1)-dimesional Minkowski spacetime with a pointlike qudit detector moving along the accelerated trajectory $(\tau) \equiv (t(\tau), x(\tau), \boldsymbol{x}_\perp(\tau))$ parametrized by proper time $\tau$, with

$$t(\tau) = \frac{1}{a}\sinh a\tau\,, \quad x(\tau) = \frac{1}{a}\cosh a\tau\,, \tag{1}$$

and $\boldsymbol{x}_\perp(\tau) = \mathbf{0}$.

We consider the interaction Hamiltonian to be

$$\hat{H}_I(\tau) = \lambda\chi(\tau)\hat{J}_x(\tau) \otimes \hat{\phi}((\tau))\,, \tag{2}$$

and the free Hamiltonian of the detector to be

$$\hat{\mathfrak{h}} = \Omega(\hat{J}_z + j)\,, \tag{3}$$

where $\hat{J}_i$ are the generic angular momentum operators for the spin-$j$ SU(2) qudit.

## Qutrit

Turning our attention first to the spin-1 qutrit, using the Dicke basis ordered as $\{|-1\rangle, |0\rangle, |1\rangle\}$, we have that the spin operators are given by

$$J_x = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \qquad J_z = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{4}$$

The detector initialized in the diagonal state with entries $a$, $b$, $c$ will look like an $X$-state

$$\hat{\rho}_{\text{D},\infty} = \begin{bmatrix} a + \rho_{11}^{(2)} & 0 & \rho_{13}^{(2)} \\ 0 & b + \rho_{22}^{(2)} & 0 \\ \rho_{13}^{(2)*} & 0 & c + \rho_{33}^{(2)} \end{bmatrix} + \mathcal{O}(\lambda^4)\,, \tag{5}$$

where

$$\begin{aligned} \rho_{11}^{(2)} &= \frac{\lambda^2}{2}\Big(b\mathcal{L}_+ - a\mathcal{L}_-\Big)\,, \\ \rho_{22}^{(2)} &= \frac{\lambda^2}{2}\Big(a\mathcal{L}_- + c\mathcal{L}_+ - b(\mathcal{L}_- + \mathcal{L}_+)\Big)\,, \\ \rho_{33}^{(2)} &= \frac{\lambda^2}{2}\Big(b\mathcal{L}_- - c\mathcal{L}_+\Big)\,, \\ \rho_{13}^{(2)} &= \frac{\lambda^2}{2}\Big(b\mathcal{I}_+ - a\mathcal{Q}_+^* - c\mathcal{Q}_-\Big)\,. \end{aligned} \tag{6}$$

We were able to show that with Gaussian switching, the off-diagonal terms vanish at late times. Thus we are left to verify the detailed balance condition.

## Important Result

The detailed balance condition is the statement that the excitation probabilities $\text{Pr}_{i\to j}$ between the $i \to j$-th energy levels (assuming $E_i < E_j$) is given by

$$\frac{\text{Pr}_{i\to j}}{\text{Pr}_{j\to i}} = e^{-\beta(E_j - E_i)}\,. \tag{7}$$

However, for this qutrit model, at leading order in perturbation theory we have

$$\text{Pr}_{-1\to 1} = \text{Pr}_{1\to -1} = 0\,, \tag{8}$$

so the detailed balance condition as defined above is *ill-defined*. We are thus unable to claim that the detector thermalizes, even when starting from an initially diagonal state.

## Higher-Dimensional Qudits

The more general pattern of qudit evolution to second-order in perturbation theory is shown by

$$\begin{bmatrix} 0 & 0 & -\sqrt{\frac{3}{2}}\mathcal{Q}_- & 0 & 0 \\ 0 & \frac{3}{2}\mathcal{L}_+ & 0 & \frac{3}{2}\mathcal{I}_+ & 0 \\ -\sqrt{\frac{3}{2}}\mathcal{Q}_-^* & 0 & -\frac{3}{2}(\mathcal{L}_+ + \mathcal{L}_-) & 0 & -\sqrt{\frac{3}{2}}\mathcal{Q}_+^* \\ 0 & \frac{3}{2}\mathcal{I}_- & 0 & \frac{3}{2}\mathcal{L}_- & 0 \\ 0 & 0 & -\sqrt{\frac{3}{2}}\mathcal{Q}_+ & 0 & 0 \end{bmatrix}, \tag{9}$$

which was initialized in the middle state (here for the spin-2 qudit). Note the checkerboard-like pattern that arises from vanishing one-point functions and how it is limited to nearest-neighbour effects along the diagonal. Our results from the qutrit case thus carry over to higher dimensional SU(2) systems.

## Conclusion

While the Unruh effect is a widely accepted phenomena of QFT, its operational demonstration has long been exemplified by two-level quantum detectors. We show here that when considering more general qudit systems, the Unruh effect is no longer as evident. In particular the detailed balance condition can no longer be utilized as a smoking gun for thermalization.

As part of this work investigating the Unruh effect for higher-dimensional quantum detectors, we are also looking into the following qutrit models:

- Heisenberg-Weyl 'clock-and-shift' qudits,
- $\Lambda$ qutrits with a degenerate ground state, and
- SU(3) 'colour' qutrits.

Even with these preliminary results, it is clear that identifying the Unruh effect in higher-dimensional quantum systems requires more care than for the case of a simple two-level system, whether a qubit proper, or a virtual qubit embedded in a higher dimension system [1].

## Additional information

Let us define a few relevant integrals which arise from our perturbative calculations:

$$\begin{aligned} \mathcal{I}_\pm &:= \int \mathrm{d}t\,\mathrm{d}t'\,\chi(t)\chi(t')e^{\pm i\Omega(t+t')}\mathsf{W}(t,t')\,, \\ \mathcal{L}_\pm &:= \int \mathrm{d}t\,\mathrm{d}t'\,\chi(t)\chi(t')e^{\pm i\Omega(t-t')}\mathsf{W}(t,t')\,, \\ \mathcal{Q}_\pm &:= \int \mathrm{d}t\,\mathrm{d}t'\,\Theta(t-t')\chi(t)\chi(t')e^{\pm i\Omega(t+t')}\mathsf{W}(t,t')\,. \end{aligned}$$

Given our detector specifications, these integrals are somewhat messy. While they might not have closed-form solutions, only the $\mathcal{Q}_\pm$ integral with the Heaviside distribution requires a UV cutoff.

We were able to identify the late-time limit of these integrals, which is required to identify thermal behaviours.

## References

[1] T. Rick Perche.
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## Contact Information

- Web: https://uwaterloo.ca/institute-for-quantum-computing/contacts/everett-patterson
- Email: ea2patterson@uwaterloo.ca