From observer-dependent facts to frame-dependent measurement records in Wigner's friend scenarios

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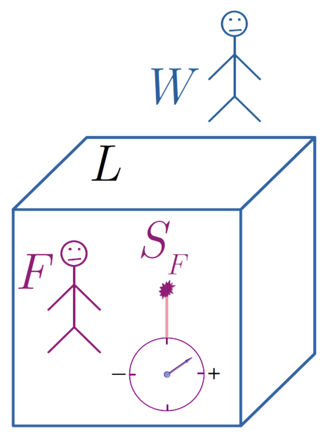
Original Wigner's friend scenario

- A thought experiment involving a super-observer W measuring an isolated laboratory in which an agent F makes a measurement on a quantum system.
- Should the super-observer update his state upon the friend's measurement, or consider the isolated laboratory as a quantum superposition evolving unitarily?

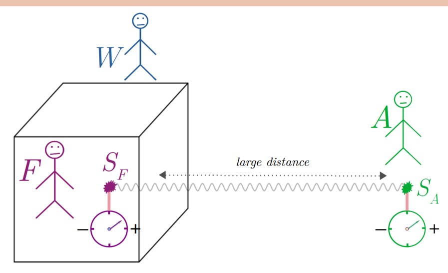


Our extended version of WF scenario

- A friend F in a sealed laboratory, a super-observer W sitting next to L, and a distant observer A (for Alice).
- A and F share an entangled state $|\psi\rangle$.

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big(|+\rangle_F\,|+\rangle_A + |-\rangle_F\,|-\rangle_A\Big)$$

$\Big(|\pm\rangle$ are the eigenstates of $\sigma_z\Big)$



Conflicting Assumptions

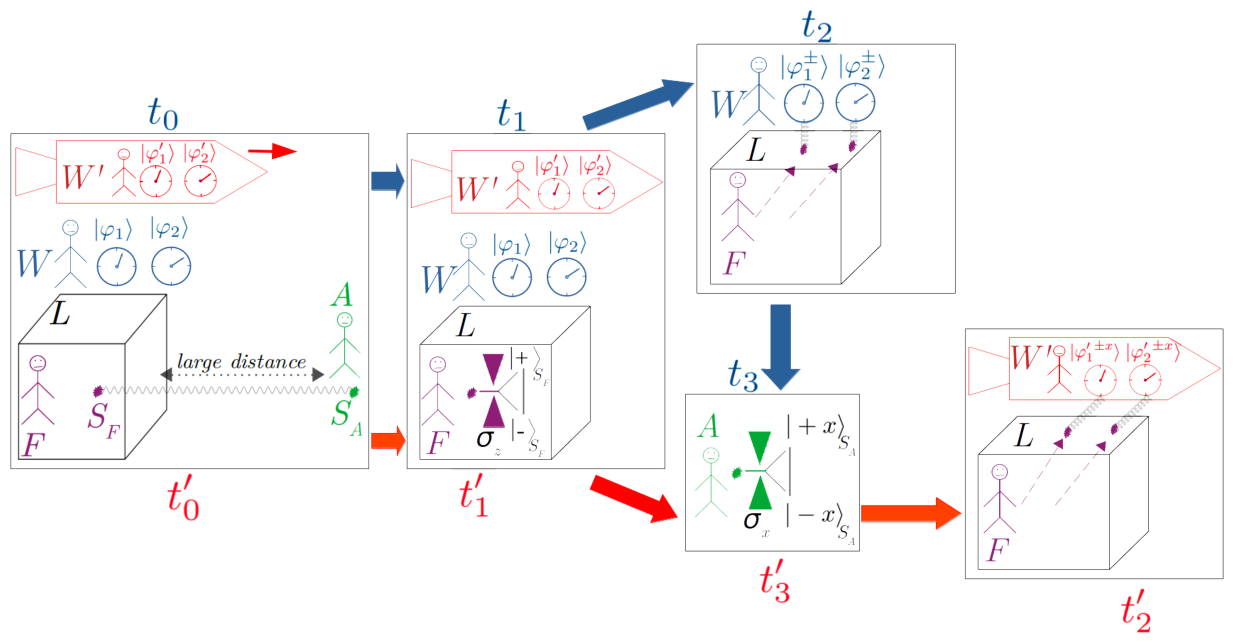
- ***Unitary evolution of the lab*** after the friend measures . Between the moments of the friend's measurement and that of Wigner, the latter describes the entire laboratory (friend, measuring device, and quantum particle) as being in a macroscopic superposition.
- ***Instantaneous update of the quantum state*** in any inertial reference frame. The current consensus is that state update can be assumed to take place instantaneously in any reference frame.

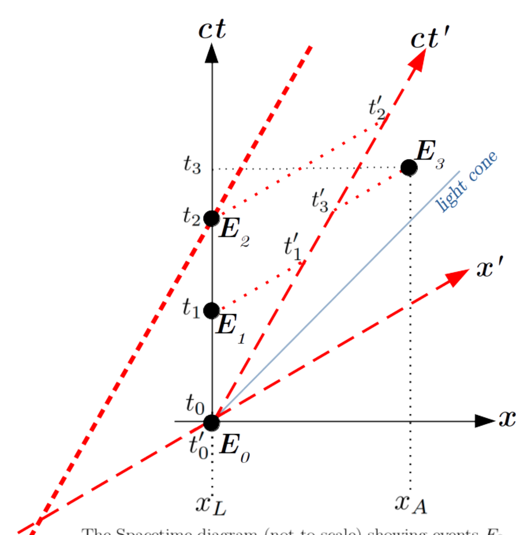
In this work, we show that the above two assumptions lead to measurement records that depend on the inertial frame of reference of the super-observer

Protocol

We consider the following 4 events in the reference frame $\mathcal{R}$, in which W is at rest, and then in frame $\mathcal{R}'$ in which another super-observer W′ is at rest. W and W′ are equipped with 2 weak pointers each, represented by the states $|\varphi_1\rangle|\varphi_2\rangle$ and $|\varphi_1'\rangle|\varphi_2'\rangle$ respectively. The time ordering of events E_2 and E_3 is inverted in $\mathcal{R}$ relative to $\mathcal{R}'$.

- $\boldsymbol{E}_0$ The system is created in state $|\Psi(t=0)\rangle = |\psi\rangle\,|m_0\rangle\,|\varepsilon_0\rangle\,|\varphi_1\rangle\,|\varphi_2\rangle$, with $|m_k\rangle$ the pointer states of the measurement apparatus, and the other degrees of freedom collectively represented by the environment states $|\varepsilon_k\rangle$.
- $\boldsymbol{E}_1$ The friend measures the spin component along z by coupling her pointer to the particle.
- $\boldsymbol{E}_2$ F creates 2 particles with the spin prepared according to the outcome she obtained and sends them outside L, where they are immediately measured by an observer (W or W′). (See weak measurement protocol in the next blocks)
- $\boldsymbol{E}_3$ A measures her qubit in the $|\pm x\rangle$ basis ($|\pm x\rangle = (|+\rangle \pm |-\rangle)/\sqrt{2}$)





The Spacetime diagram (not to scale) showing events E_0-E_3 with respect to reference frames $\mathcal{R}$ (solid axes (ct,x)) and $\mathcal{R}'$ (dashed axes(ct', x')). The time of event E_i in each reference frame is shown along with the space-like hypersurface in that reference frame (dotted lines). The positions of x_L and x_A of the lab L and agent A respectively are also shown, as well as the position of W′, fixed in $\mathcal{R}'$.

Weak measurement in $\mathcal{R}$

- The friend measures her spin at t_1, the state becomes

$$|\Psi(t_1)\rangle = \frac{1}{\sqrt{2}}\Big(|L_+\rangle_F\,|+\rangle_A + |L_-\rangle_F\,|-\rangle_A\Big)\,|\varphi_1\rangle\,|\varphi_2\rangle$$

$|L_\pm\rangle \equiv |\pm\rangle_F\,|m_\pm\rangle\,|\varepsilon_\pm\rangle$ denotes the quantum state of the entire isolated laboratory.

- At t_2, F creates two qubits and sends them outside L to be weakly measured by W.

$$|\Psi(t_2)\rangle = \frac{1}{\sqrt{2}}\Big(|L_+\rangle_F\,|+\rangle\,|+\rangle\,|+\rangle_A + |L_-\rangle_F\,|-\rangle\,|-\rangle\,|-\rangle_A\Big)\,|\varphi_1\rangle\,|\varphi_2\rangle$$

- W immediately makes a weak measurement of the two qubits: $e^{(-ig\sigma_z P_1)}e^{(-ig\sigma_z P_2)}\,|\Psi(t_2)\rangle$.
- Post selection: $\langle +\theta_1|\,\langle +\theta_2|\,\Psi(t_2)\rangle$. ($|+\theta_i\rangle$ is the positive eigenstate of σ_{θ_i})
- After post-selection W measures the position X_1 and X_2 of each pointer. By repeating the experiment, he collects statistics to determine the average pointer positions. The average is obtained from $|\Psi(t_2)\rangle\langle\Psi(t_2)|$ by tracing out the friend's and Alice's degrees of freedom in order to obtain the reduced density matrix ρ_{12}. Using $\Pi_{+\theta_i} \equiv |+\theta_i\rangle\,\langle+\theta_i|$, W obtains

Average in $\mathcal{R}$

$$\langle X_1 X_2\rangle = \mathrm{Tr}\left(\rho_{12}\Pi_{+\theta_1}\Pi_{+\theta_2}X_1X_2\right) = \frac{g^2}{4}\left(1+\cos\theta_1\cos\theta_2\right).$$

Weak measurement in $\mathcal{R}'$

In $\mathcal{R}'$, E_3 takes place before E_2

- At t_3', Alice measures her qubit in the $|\pm x\rangle$ basis.

$$|\Psi'(t_1')\rangle \rightarrow \begin{cases} |\Psi'_{+x}(t_3')\rangle = \frac{1}{\sqrt{2}}\left(|L_+\rangle + |L_-\rangle\right) \equiv |L_{+x}\rangle \\ |\Psi'_{-x}(t_3')\rangle = \frac{1}{\sqrt{2}}\left(|L_+\rangle - |L_-\rangle\right) \equiv |L_{-x}\rangle \end{cases}$$

- At t_2' F creates 2 qubits and sends them outside to be weakly measured by W′.

$$\begin{cases} |\Psi'_{+x}(t_2')\rangle = |L_{+x}\rangle\,|+x\rangle\,|+x\rangle\,|\varphi_1'\rangle|\varphi_2'\rangle \\ |\Psi'_{-x}(t_2')\rangle = |L_{-x}\rangle\,|-x\rangle\,|-x\rangle\,|\varphi_1'\rangle|\varphi_2'\rangle \end{cases}$$

- The density matrix $\rho(t_2')$ can be obtained from the above equation for $|\Psi'(t_2')\rangle$.
- After many runs of the experiment W′ obtains

Average in $\mathcal{R}'$

$$\langle X_1' X_2'\rangle = g^2\cos\theta_1\cos\theta_2$$

Observers in $\mathcal{R}$ and $\mathcal{R}'$ will disagree on the average pointer shifts obtained by measuring the qubits.

A modified scenario: projective measurement

- **In $\mathcal{R}$**:
  - At t_2 F sends her 2 qubits outside the lab.
  - W makes a projective measurement in the $|\pm\rangle$ basis on one qubit, and in the $|\pm x\rangle$ basis on the other.
  - The lab state is updated to $|L_+\rangle$ or $|L_-\rangle$
  - F declares her physical record to be $+$ or $-$.
- **In $\mathcal{R}'$**:
  - At t_3' A measures in the $|\pm x\rangle$ basis, updating the lab to $|L_{+x}\rangle$ or $|L_{-x}\rangle$
  - At t_2', F creates 2 qubits in state $|\pm x\rangle$ and sends them outside the lab.
  - W′ measures one qubit in the $|\pm\rangle$ basis and the other in the $|\pm x\rangle$ basis.
  - F can now release her records, which can be verified by W′ to be $+x$ or $-x$.

This modified protocol leads to different physical records being observed in $\mathcal{R}$ and $\mathcal{R}'$.

Conclusion

- Assuming the friend and Wigner observe different facts also leads to observers in different reference frames disagreeing on their observations.
- While an isolated macroscopic system might well be accounted for unitarily, demanding an agent's arbitrary operations to be described with unitaries implies stronger constraints.
  - Can we model the agent's operations by assuming perfect correlations between the state of the measured spin and the state of the lab (without relying on interventions)?
  - Can we account for the entanglement of a massive, complex system (the lab) as we do for an elementary particle or a photon?

References:

- E. P. Wigner, in The Scientist Speculates edited by I. J. Good (Heinemann, 1962)
- J. Allam, A. Matzkin, arXiv:2304.09289 (2023) (and references therein)